ON PECULIARITIES OF THE ANISOTROPIC DIFFUSION DURING FORBUSH EFFECTS OF GALACTIC COSMIC RAYS

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Experimental data of neutron super monitors, solar wind velocity and components of the interplanetary magnetic field (IMF) have been used to study a relationship between the temporal changes of the energy spectrum of the Forbush effects of galactic cosmic rays (GCR) and the power spectral density (PSD) of the IMF’s strength fluctuations. Based on the energy spectrum of the Forbush effects of GCR a structure of the IMF’s fluctuations is determined in the disturbed vicinity of the interplanetary space when the direct (in situ) measurements of the IMF are absent. In order to study anisotropic diffusion propagation of GCR a second order four dimensional Fokker–Plank’s type partial differential equation has been numerically solved. Diffusion, convection, drift due to the regular component of the IMF and adiabatic energy changes of the GCR particles because of the interaction with the diverged solar wind inhomogeneities are included in the transport equation. The spatial distributions of the density, radial, heliolatitudinal and heliolongitudinal gradients during the Forbush effect of GCR intensity have been found for the positive (qA > 0) period of solar magnetic cycle. It is shown that a stationary diffusion–convection–drift approximation of GCR transport is an acceptable model for describing the recurrent Forbush effects of GCR associated with the established corotating disturbances in the inner heliosphere.

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1. Introduction

High precision experimental data of the solar wind density, temperature, velocity and IMF have become available owing to different space probes, among which is the SOHO instrument launched in 1996 [1]. The SOHO instrument is located at the Langrangean point L1 on the Sun–Earth line, 1.5 \times 10^6 \text{ km} away from the Earth. So, unique measurements on the SOHO board are not undergoing any influence of the Earth magnetosphere. Generally instruments on the board of space probes (Pioneer, Voyager, Ulysses, SOHO and so on) have supplied data which suggest organized large-scale fluctuations in heliospheric plasma, despite a single interplanetary probe data do not provide enough information about the spatial topological structure of the electro-magnetic fields in the interplanetary space. From time to time, at the maxima epochs of solar activity in the interplanetary space, there are observed disturbances (shock waves, magnetic clouds and other types of structures after the coronal mass ejecta (CME) and solar flares) with drastically large range changes of the measurable parameters. Unfortunately, in these cases in situ data of the above mentioned parameters are often absent due to the instrumental restrictions of the spacecraft. As an example there can be noted the situation that took place recently on the October 29/30, 2003. For this period main sensor of the SOHO instrument (after the outstanding CME) indicated that the solar wind speed exceeded the maximum speed detectable by the Proton Monitor (\(\sim 1100 \text{ km/s}\)) [1].

As a rule, substantial disturbances in the interplanetary space are accompanied by the short period decreases of the GCR intensity (Forbush effects). It was shown [2–5] that Forbush effects of GCR intensity observed by neutron monitors, ground and underground meson telescopes can provide basically useful information about the structure of the IMF’s fluctuations. Particularly, it gives an opportunity to determine the temporal changes of the PSD of the IMF’s strength fluctuations in the disturbed vicinity of the interplanetary space being generally responsible for the scenario of GCR diffusion in course of Forbush effect.

This paper is devoted to the problems partially discussed in our previous papers [4–5]. Namely, the goal is to find a relationship between the exponent \(\nu\) of the PSD of the IMF fluctuations (PSD \(\propto f^{-\nu}\), where \(f\) is frequency) and the energy spectrum exponent \(\gamma\) of the Forbush effects of GCR (\(\delta D(R)/D(R) \propto R^{-\gamma}\), where \(R\) is the GCR particle’s rigidity) in two cases; first, when there exist simultaneous data of the IMF’s components and GCR intensity and the second, data of GCR intensity are available, but data of the IMF’s components are absent. In the last case, an exponent \(\nu\) of the PSD of the IMF fluctuations was found using the energy spectrum exponent \(\gamma\) of the Forbush effects of GCR [2–5]. Next issue is to find the
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Spatial distributions of density, radial, latitudinal and azimuthal gradients of GCR based on the solution of GCR transport equation using the generalized tensor of anisotropic diffusion for the positive \((qA > 0)\) period of solar magnetic cycle.

2. Experimental results and discussion

Data of neutron super monitors and the IMF’s \(B_x\), \(B_y\), and \(B_z\) components have been used to study peculiarities of the rigidity spectrum of the GCR intensity variations for the recurrent and sporadic Forbush effects [4,5]. The power type rigidity spectrum of the GCR intensity variations \((\delta D(R)/D(R) \propto R^{-\gamma})\) have been calculated according to the method given in [6,7]

\[
\frac{\delta D(R)}{D(R)} = \begin{cases} 
AR^{-\gamma} & \text{for } R \leq R_{\text{max}}, \\
0 & \text{for } R > R_{\text{max}},
\end{cases}
\]

where \(R\) is rigidity, \(A\) is power of the rigidity spectrum and \(R_{\text{max}}\) is upper limiting rigidity beyond which the Forbush effect of GCR intensity vanishes. Results of calculations of the rigidity spectrum exponent \(\gamma\) of the sporadic Forbush effect for the period 7–29 July 2000 (Fig. 1(a)) using data of the super neutron monitors (Apatity, Calgary, Climax, Haleakala, Kiel, Lomnicky Stit, Mexico, Moscow, Oulu, Rome, Thule) and the method presented in [6,7] are shown in Fig. 1(b).

Fig. 1. Temporal changes: (a) of the GCR intensity for the period 7–29 July 2000, M — Mexico City, O — Oulu, C — Calgary; error bars on observations are comparable to size of data curves width and hence omitted; (b) of the rigidity spectrum of this Forbush effect.

It is seen from the Fig. 1(b) that the rigidity spectrum of the sporadic Forbush effect is relatively soft \((\gamma = 1.4)\) at the beginning phase of the decreasing of GCR intensity, then it is gradually hardening before reaching the
minimum level of the intensity \((\gamma = 0.2)\). After it, during 4–5 days of the recovery period the rigidity spectrum is progressively softening \((\gamma = 0.8–0.9)\), and then \(\gamma\) remains constant for 9–10 days. This kind of changes of the rigidity spectrum of the outstanding sporadic Forbush effect of GCR (7–29 July 2000) can be ascribed to the specific changes of the IMF’s irregularities structure in the vicinity of the interplanetary space. Specifically, at the beginning phase of the GCR intensity decrease, the vicinity of the disturbances in the interplanetary space is in the stage of developing and is, generally, undergoing the spatial extension. The velocity of the disturbances (shock waves or magnetic clouds) is large and an intensive convection of the relatively low energy particles of GCR takes place. Owing to the above mentioned reasons relatively low energy particles of GCR are preferentially modulated and \(\gamma\) is large \((\gamma = 1.4)\), i.e. rigidity spectrum of the Forbush effect is soft. Then due to the extension of the size of the disturbances an intensive interaction of the relatively high velocity disturbances with the background solar wind takes place. This interaction leads to the creation of the additional large size irregularities of the IMF [5, 7]. At the same time, a variety of the structure of the fluctuations of the IMF (being rather different for the various cases) directly connected with the peculiarities of the origin of the disturbances on the Sun [8] must be taken into account, too. Due to above mentioned reasons in this phase of the Forbush effect there should be preferentially modulated relatively higher energy particles of GCR than before. These should result in the hard rigidity spectrum of the Forbush effects of GCR, which is really observed \((\gamma = 0.2–0.4)\) in the experimental data of neutron monitors. Finally, the Earth gradually leaves the region of the disturbances and the energy spectrum of the Forbush effect slowly becomes soft. The above mentioned explanation of the energy spectrum of the Forbush effect is dealing with the case of GCR modulation when generally a diffusion-convection model is valid. This kind of scenario takes place for many cases of Forbush effect of the sporadic type when the disturbances occupy a large size of the inner heliosphere [4, 7]. However, there have been observed some sporadic Forbush effects of GCR with the peculiar behavior directly connected with the individual geometrical (topological) structure of the shock waves, magnetic clouds and intensity of the CME [9, 10].

There also exists an interesting problem concerned with the recurrent Forbush effect of GCR connected with the established corotating (with a period of 27 days) disturbances in the interplanetary space. Unfortunately, a separation problem of the recurrent Forbush effects and the 27-day variations of GCR is not solved up to present. The point is that the observation of GCR intensity simultaneously contains both types of variations. So, for the separation of the recurrent Forbush effects and the 27-day variation of GCR additional information is needed. For this case valuable information can
be obtained from data of the specific temporal changes of the solar wind velocity and IMF playing an important role in the creation of the recurrent Forbush effect of GCR.

In Fig. 2 there are presented temporal changes of the magnitude $B$ of the IMF [11], the solar wind velocity $U$ [12] and the GCR intensity $N$ [13] for the period of 15 June–29 August 1994. It is seen from these figures that in GCR intensity $N$ there is observed a clear $\sim 14$ days periodicity negatively correlated with the solar wind velocity $U$ (correlation coefficient $r$ is equal to $\sim -0.74 \pm 0.05$). At the same time there is not any direct correlation between $N$ and $B$ ($r \sim -0.14$). However, $r \sim -0.51 \pm 0.07$ when data of IMF is shifted for two days with respect to GCR intensity $N$. It means that for the above mentioned period the changes of the IMF are reflected in the changes of GCR intensity two days later. One can conclude that periods of GCR intensity changes on 15–25 June 1994, 11–25 July 1994 and 6–20 August 1994 can be considered as the recurrent Forbush effects according to the behavior of the solar wind velocity $U$ and the strength $B$ of the IMF.

Fig. 2. Temporal changes of the magnitude $B$ of the IMF from ACE [11], the solar wind velocity $U$ from OMNI [12] and the GCR intensity $N$ [13] of Moscow neutron monitor for the period of 15 June–29 August 1994.
In Fig. 3 are presented the temporal changes of the GCR intensities for Kiel and Deep River neutron monitors (Fig. 3(a)) and the rigidity spectrum exponent $\gamma$ in the course of the Forbush effect of 6–20 August 1994 (Fig. 3(b)). It must be stressed that the reliability of the rigidity spectrum exponent $\gamma$ is an important problem when the amplitude of Forbush effect is small. In connection with this in Fig. 3(b) the results for the period of 9–14 August 1994, when amplitudes of the Forbush effect were greater than 0.5% are presented. Unfortunately, for the recurrent Forbush decreases this dilemma often exists. However, $\gamma$ can be calculated reasonably reliable due to sufficient number of neutron monitors. It is seen from Fig. 3(b) that for the considered recurrent Forbush effect (6–20 August 1994) the rigidity spectrum of the Forbush effect (based on the Alma-Ata, Apatity, Beijing, Climax, Deep River, Hermanus, Kiev, Lomnicky Stit, Mexico, Moscow neutron monitors data) generally is hard with some insignificant changes during the course of the Forbush effect. Particularly, one can recognize an unimportant tendency of the hardening of the energy spectrum; $\gamma = 0.5$ at the beginning phase of the Forbush effect and $\gamma = 0.3$ near the minimum of GCR intensity. It is noteworthy to state that the recurrent disturbances are well established phenomenon and can exist during 3–5 rotations of the Sun. The Earth crosses the recurrent disturbances with linear velocity $\sim 400$ km/s and the peculiarities of the Forbush effect among other reasons depend on the geometrical size of this disturbance, too. The range of the heliolatitude angle $\varphi$ of the disturbances responsible for the considered recurrent Forbush effect is $\sim (70^\circ - 80^\circ)$ and it corresponds to the distance of $2 \times 10^{13}$ cm on the Earth orbit. This is one of the reasons why the duration of the considered Forbush effect is relatively short. For this Forbush effect with the hard rigidity spectrum there should be noted that it has taken place on

![Fig. 3. Temporal changes: (a) of the GCR intensities for period 6–20 August 1994, K — Kiel, D — Deep River; error bars on observations are comparable to size of data curve width and hence omitted. (b) of the rigidity spectrum of this Forbush effect.](image-url)
the background of the well established long period (few months) recurrent disturbances and rigidity spectrum of GCR intensity during this period generally was hard. This effect is also pronounced in the rigidity spectrum of GCR Forbush effect.

For the diffusion-convection approximation of GCR transport an exponent \( \gamma \) of the rigidity spectrum of the isotropic intensity variations of GCR for the energy more than 10 GeV (11-year variation) is determined by the parameter \( \alpha \) showing the dependence of the diffusion coefficient \( K \) on the rigidity \( R \) of GCR, \( K \propto R^\alpha \) [3–5,14,15]. The same can be stated when in the course of the Forbush effect a diffusion-convection approximation is valid [4,5]. So, temporal (from day to day) changes of the rigidity spectrum of the Forbush effect of GCR give an opportunity to recognize the structural evolution in time of the IMF’s irregularities of that part of the interplanetary space, which is occupied by the disturbances (vicinity of the extended shock waves, magnetic clouds etc). This is one kind of the inverse problems of GCR variations. In the case of the diffusion–convection approximation’s scenario, the rigidity spectrum of the Forbush effect of GCR can be used as a source of the information concerning the disturbed vicinity of the interplanetary medium; particularly, basing on the energy spectrum of the Forbush effects of GCR it is possible to find the exponent \( \nu \) of the PSD of the IMF’s fluctuations (PSD \( \propto f^{-\nu} \), where \( f \) is the frequency), when direct (in situ) measurements of the IMF are absent in the disturbed vicinity of the interplanetary space. Bearing in mind that for the finding of a reliable value of the exponent \( \nu \) there are not enough experimental data (e.g. for one day period) of the IMF’s irregularities during the Forbush effects, data of GCR intensity become very valuable and unique. Moreover, this method can be used to find an exponent \( \nu \) of PSD of the IMF’s fluctuations for the arbitrarily short period. The range of this period would be determined only by the suitable statistics of the GCR intensity measurements. It is known from [14–17] that the parameter \( \alpha \) depends on the IMF’s structure and changes over the range from zero to 2, \((0 \leq \alpha \leq 2)\) and depends on the exponent \( \nu \) as \( \alpha = 2 - \nu \). In [4–6] it was shown that there is direct relationship between \( \gamma \) and \( \alpha \), so one can write, \( \gamma = 2 - \nu \). Therefore, using the temporal changes of the rigidity spectrum exponent \( \gamma \) of the GCR isotropic intensity variations for the determination of the structure of the IMF irregularities during the Forbush effects is very perspective.

Fortunately, for the powerful Forbush effect of 7–29 July 2000 there is sufficient statistics for finding the reliable relationship between \( \gamma \) and \( \nu \). Indeed, during 13–14 days (15–29 July 2000) the daily rigidity spectrum exponent \( \gamma \) of the Forbush effect is constant in the scope of the accuracy and for this period the PSD of the IMF’s fluctuations based on 5-minute data [11] can be found. In figure 4 there are presented PSD of the \( B_x \),
$B_y$ and $B_z$ components of the IMF’s strength fluctuations for the period of 15–29 July 2000. Assuming a fairly accurate relationship between $\gamma$ and $\nu$ ($\gamma \approx 2 - \nu$) in the table there are presented the values of $\nu$ obtained from the temporal changes of the rigidity spectrum exponent $\gamma$ of the Forbush effect for each day of the period 8–14 July 2000; in this table (last column) there are presented the values of $\nu$ obtained from the in situ measurements data of IMF for the period of 15–29 July 2000 and the average value of $\gamma$ corresponding to this period.

<table>
<thead>
<tr>
<th>Day</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15-29</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.30±0.07</td>
<td>0.8±0.07</td>
<td>0.8±0.07</td>
<td>0.7±0.07</td>
<td>0.2±0.11</td>
<td>0.37±0.11</td>
<td>0.47±0.09</td>
<td>0.8±0.02</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.8±0.07</td>
<td>1.1±0.07</td>
<td>1.1±0.07</td>
<td>1.2±0.07</td>
<td>1.8±0.11</td>
<td>1.6±0.11</td>
<td>1.5±0.09</td>
<td>0.9±0.12</td>
</tr>
</tbody>
</table>

Taking into account the accuracies of the calculations of $\gamma$ and $\nu$ (last column in the table) there are expected two extreme values 1.63 and 1.87 of $\gamma + \nu$. It means that for the period of 15–29 July 2000 the above mentioned expected relationship $\gamma + \nu \approx 2$ is fulfilled with the average accuracy of 10–12%. Thus, the existence of the reliably established relationship between the rigidity spectrum exponent $\gamma$ of the Forbush effect and the exponent $\nu$ of the PSD of the IMF’s irregularities gives an opportunity to obtain an important information concerning the almost instantaneous structure of the IMF’s fluctuations (for the period much shorter than one day). Time-span of the minimal period can be determined by the statistical accuracies of the GCR intensity data being reasonable for the reliable calculations of the rigidity spectrum exponent $\gamma$ of the GCR intensity variations.
3. Theoretical model of the recurrent Forbush effect of GCR

Generally, the short period change of the GCR intensity is a non stationary process and for its describing a time-dependent transport equation must be considered. An initial phase of the powerful sporadic type Forbush effects belongs to these types of phenomena, as the period shorter than one day a significant decrease of the GCR intensity is observed; at the same time the changes of GCR intensity during the recovery period of the sporadic Forbush effect (lasting a few days) in some approximation can be considered as a quasi stationary process. The same can be stated concerning the great majority of the GCR recurrent Forbush effects. The last statement is justified by the fact that the amplitude of the recurrent Forbush effect of GCR intensity is rather small (≤ 2%) and duration is reasonably large (7–10 days).

The recurrent Forbush effect of GCR takes place due to well established co-rotating heliolongitudinal disturbances in the interplanetary space. The Earth crosses these heliolongitudinal disturbances and neutron monitors are registering the Forbush effect of GCR. The behavior of the GCR intensity during the recurrent Forbush effect is described by transport equation [18,19]. In the spherical \( r, \theta, \varphi \) coordinate system the equation has the following form

\[
\frac{\partial N}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( K_{rr} \frac{\partial N}{\partial r} + K_{r\theta} \frac{1}{r} \frac{\partial N}{\partial \theta} + K_{r\varphi} \frac{1}{r \sin \theta} \frac{\partial N}{\partial \varphi} \right) \right] \\
+ \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \left( K_{\theta r} \frac{\partial N}{\partial r} + K_{\theta \theta} \frac{1}{r} \frac{\partial N}{\partial \theta} + K_{\theta \varphi} \frac{1}{r \sin \theta} \frac{\partial N}{\partial \varphi} \right) \right] \\
+ \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \left[ K_{\varphi r} \frac{\partial N}{\partial r} + K_{\varphi \theta} \frac{1}{r} \frac{\partial N}{\partial \theta} + K_{\varphi \varphi} \frac{1}{r \sin \theta} \frac{\partial N}{\partial \varphi} \right] \\
- \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 NU_r \right] - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta NU_\theta \right] - \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \left[ NU_\varphi \right] \\
+ \left( \frac{R}{3} \frac{\partial N}{\partial R} + \frac{n}{3} \right) \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 U_r \right] + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta U_\theta \right] + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} U_\varphi \right). \tag{2}
\]

Diffusion coefficients \( K_{rr}, K_{r\theta}, \ldots, K_{\varphi\varphi} \) are components of the generalized anisotropic diffusion tensor for the 3 dimensional \((B_r, B_\theta, B_\varphi)\) IMF [2,4]; among these 9 components, e.g. two components \( K_{rr}, K_{r\theta} \) have the form

\[
K_{rr} = K_{rr} [\cos^2 \delta \cos^2 \psi + \beta (\cos^2 \delta \sin^2 \psi + \sin^2 \delta)], \\
K_{r\theta} = K_{r\theta} [\sin \delta \cos \delta \cos^2 \psi(1 - \beta) - \beta \sin \psi], \tag{3}
\]

where \( \delta \) is the angle between the lines of the IMF and the radial direction in the meridian plane \( (\delta = \arctan(B_\theta/B_r)) \) and \( \psi \) is the angle between the lines of the IMF and radial direction from the Sun in the constant heliolatitude.
plane \((\psi = \arctan(-B_\varphi/B_r))\); \(\beta = K_\perp/K_\parallel\) and \(\beta_1 = K_d/K_\parallel\); \(K_\parallel, K_\perp\) and \(K_d\) are parallel, perpendicular and drift diffusion coefficients with respect to the IMF lines, respectively. In the considered model of the GCR transport the following assumptions are made: for the recurrent Forbush effect the processes in the interplanetary space is assumed to be a quasi stationary and the term \(\frac{\partial N}{\partial t}\) in Eq. (2) can be neglected

- \(\frac{\partial N}{\partial t} = 0\);
- \(K_\parallel = K_0K(r)K(\theta, \varphi)K(R)\); \(K_0 = 4 \times 10^{22}\text{cm}^2/\text{s}; K(R) = R^{0.5}\);
- \(K(r) = 1 + 0.5\left(\frac{r}{r_+}\right)\) where \(r_+\) is a distance between the Sun and the Earth;
- \(K(\theta, \varphi) = 1 - 0.4\sin(3(\varphi - 75^\circ))\sin^6\theta\);
- \(U = U_0U(\theta, \varphi)\), where \(U(\theta, \varphi) = 1 + 0.9\sin(3(\varphi - 75^\circ))\sin^6\theta\);
- \(U_0 = 500\text{ km/s}\.\)

In our model \(K(\theta, \varphi)\) and \(U(\theta, \varphi)\) describe the disturbances in the ranges of \(\varphi = (75-135)^\circ\) and \(\theta = (60-120)^\circ\) (Fig. 5).

\[\frac{\partial^2 n}{\partial r^2} + A_2 \frac{\partial^2 n}{\partial \theta^2} + A_3 \frac{\partial^2 n}{\partial \varphi^2} + A_4 \frac{\partial^2 n}{\partial r \partial \theta} + A_5 \frac{\partial^2 n}{\partial r \partial \varphi} + A_6 \frac{\partial n}{\partial r} + A_7 \frac{\partial n}{\partial \theta} + A_8 \frac{\partial n}{\partial \varphi} + A_9 n + A_{10} \frac{\partial n}{\partial R} = 0.\]  

Fig. 5. Changes of the parameters \(U(\theta, \varphi)\) and \(K(\theta, \varphi)\) versus heliolongitude for \(\theta = 90^\circ\).

The free parameter \(K_0\) gives an opportunity to adjust the solutions to the experimental results of the neutron monitors data.

Taking into account the above mentioned expressions Eq. (2) is reduced to the form

\[A_1 \frac{\partial^2 n}{\partial r^2} + A_2 \frac{\partial^2 n}{\partial \theta^2} + A_3 \frac{\partial^2 n}{\partial \varphi^2} + A_4 \frac{\partial^2 n}{\partial r \partial \theta} + A_5 \frac{\partial^2 n}{\partial r \partial \varphi} + A_6 \frac{\partial n}{\partial r} + A_7 \frac{\partial n}{\partial \theta} + A_8 \frac{\partial n}{\partial \varphi} + A_9 n + A_{10} \frac{\partial n}{\partial R} = 0.\]  

(4)
This is the parabolic second order linear partial differential equation. In Eq. (4) \( n = N/N_0 \) is the dimensionless relative density, \( N \) and \( N_0 \) are density in the interplanetary space and in the galaxy, respectively. Coefficients \( A_1, A_2, \ldots, A_{11} \) are functions of the spherical coordinates \( r, \theta, \varphi \) and the GCR particle rigidity \( R \). Eq. (4) was solved numerically for two dimensional IMF [4,5] using the finite difference method. The number of equations in the linear algebraic system is equal to \( N \times M \times L = 49 \times 41 \times 360 = 723240 \), i.e. there are: 49 steps in the distance \( r \) \( (r = i \times h_1, \ h_1 \) is an alternating step and \( i = 1, 2, \ldots, N \) ), 41 steps in the zenith angle \( \theta \) \( (\theta = j \times h_2, \ h_2 \) is an alternating step and \( j = 1, 2, \ldots, M \) ) and 360 steps in the azimuthal angle \( \varphi \) \( (j = k \times h_3, \ h_3 \) is the constant step of one degree and \( k = 1, 2, \ldots, L \) ). Due to the alternating steps in \( r \) and \( \theta \) grid is denser in the distances shorter than 10 \( AU \) and near the helioequatorial regions. Boundary conditions have the following expressions

\[
\begin{align*}
\frac{\partial n}{\partial r}|_{r=0} &= 0, & n|_{r=1} &= 1, & \frac{\partial n}{\partial \theta}|_{\theta=0} = \frac{\partial n}{\partial \theta}|_{\theta=180^\circ} &= 0, \\
n|_{R=150 \text{ GV}} &= 1, & n|_{\varphi_1} &= n|_{\varphi_{L+1}}, & n|_{\varphi_{-1}} &= n|_{\varphi_{L-1}}.
\end{align*}
\]

(5)

Fig. 6. Changes of the expected density (A%) amplitude of the Forbush effect of GCR for the energy of 10 GeV.

According to the realized model of the GCR transport the amplitude of the Forbush effect for 10 GeV energy of GCR is less than 2% in good agreement with the neutron monitors experimental data (Fig. 3(a)). In Fig. 7 the expected distributions of the radial, latitudinal and heliolongitudinal gradients of GCR density are presented.

It is seen from these figures that the radial gradient of the GCR density versus heliolongitudes increases and reaches its maximum level at the minimum stage of the Forbush effect. At the same time the heliolongitudinal and latitudinal gradients of the GCR density are of the alternating sign; they have maximal magnitudes during the descending (GCR density decreases)
Fig. 7. Spatial gradients of the GCR density (a) radial, (b) latitudinal and (c) heliolongitudinal.

...and ascending (GCR density increases) phases and zero magnitudes at the moment of the minimum of the Forbush effect. The changes of the spatial gradients generally are well reflected in the anisotropy of GCR [20]. In the cases of the Forbush affects extreme changes of the spatial gradients usually are the reasons for the anomaly behavior of the GCR anisotropy [21].

4. Conclusion

1. Method for the determination of the exponent $\nu$ of the PSD of the IMF’s irregularities based on the rigidity spectrum exponent $\gamma$ of the GCR Forbush effect has been developed.

2. For the describing of the recurrent Forbush effect a diffusion–convective–drift model of the GCR transport is a fairly good approximation.

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